Period 1, March 17, 2025

4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated

$0 = 2(1) - 2$

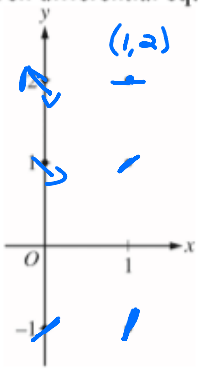
$1 = 2(1) - 1$

$3 = 2(1) - (-1)$

$1 = 2(0) - (-1)$

$-1 = 2(0) - 1$

$-2 = 2(0) - 2$



$\frac{dy}{dx} = 2x - y$

$\frac{d^2y}{dx^2} = 2(1) - \frac{dy}{dx} = 2 - (2x - y) = 2 - 2x + y =$

(2,3)

$F(2) = 3$

$\frac{dy}{dx} = 2x - y$

$= 2(2) - 3$

$+1 = 4 - 3$

increasing

$2 - 2(2) + 3$

$2 - 4 + 3 = +1$

concave up

$\frac{dy}{dx} = x^2 y \cdot dx$

$\frac{dy}{y} = x^2 dx$

$\ln a = b$

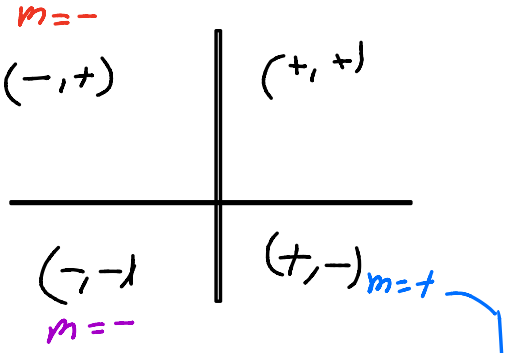
$e^b = a$

$\int \frac{1}{y}\,dy = \int x^2\,dx$

$\ln y = \frac{1}{3}x^3 + C \Rightarrow y = e^{\frac{1}{3}x^3 + C} = e^{\frac{1}{3}x^3} \cdot e^C = C_1 e^{\frac{1}{3}x^3}$

$y = C_1 e^{\frac{1}{3}x^3}$

$\frac{dy}{dx} = C \cdot e^{\frac{1}{3}x^3} \cdot x^2 = Cx^2 e^{\frac{1}{3}x^3} = yx^2$



Shown above is a slope field for which of the following differential equations?

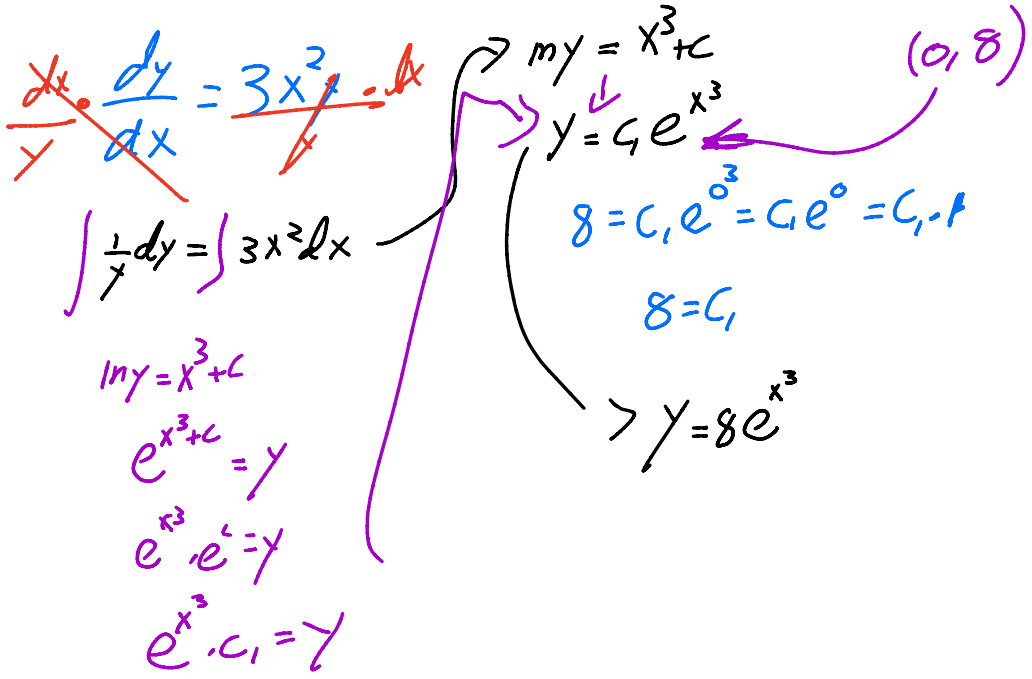
(A) $\frac{dy}{dx} = \frac{x}{y}$ ✓

(B) $\frac{dy}{dx} = \frac{x^2}{y^2}$

(C) $\frac{dy}{dx} = \frac{x^3}{y}$ ✓

(D) $\frac{dy}{dx} = \frac{x^2}{y}$

(E) $\frac{dy}{dx} = \frac{x^3}{y^2}$ ✓ ✓


$\frac{dx}{y}\cdot\frac{dy}{dx}=3x^2\cdot dx$
$\int\frac{1}{y}dy=\int 3x^2dx$
$\ln y=x^3+C$
$e^{x^3+C}=y$
$e^{x^3}\cdot e^C=y$
$e^{x^3}\cdot C_1=y$
$\ln y=x^3+C$
$y=C_1e^{x^3}$
$(0,8)$
$8=C_1e^{0^3}=C_1e^0=C_1\cdot 1$
$8=C_1$
$y=8e^{x^3}$